UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the May/June 2010 question paper

for the guidance of teachers

4037 ADDITIONAL MATHEMATICS

4037/11 Paper 11, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2010 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1,2 This is deducted from A or B marks when essential working is omitted.

PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1	**(i)** $\frac{1}{2}3x^2(1+x^3)^{-\frac{1}{2}}$	B1,B1 [2]	B1 for $\frac{1}{2}(1+x^3)^{-\frac{1}{2}}$ B1 for $\times 3x^2$
	(ii) $2x\cos 2x - 2x^2\sin 2x$	M1 A2,1,0 [3]	M1 for attempt to differentiate a product –1 each error
2	**(i)** $1 + 18x + 135x^2\ldots$	B1,B1 [2]	B2, 1, 0 –1 for each error
	(ii) $(1 \times -5) + (18 \times -3) + (135 \times 1)$ $= 76$	M1,A1ft A1 [3]	M1 for a correct method using their **(i)** A1ft on their 3 terms unsimplified
3	$(k-2)^2 - 4(2k-4)$ $k^2 - 12k + 20 = 0$ critical values 2 and 10 $k \leqslant 2$ and $k \geqslant 10$	M1 A1 M1 A1 A1 [5]	M1 for use of discriminant for 3 term quadratic in k M1 for attempt to solve quadratic A1 for critical values A1 for range
4	**(i), (ii)** and **(iii)**	B1 B1 B1 [3]	B1 for each correct Venn diagram
	(b) (i) {9,10,11,12,13,14} **(ii)** {5,6,7,8,9,10,11,12,13,14,15,16,17,18,19, 20} **(iii)** $\varnothing$ or { }	B1 B1 B1 [3]	Or equivalent Or equivalent
5	$3x^3 + 17x^2 + 18x - 8 = 0$ f(–2) = 0 (or other roots) $(x+2)(3x^2 + 11x - 4)(= 0)$ $(x+2)(3x-1)(x+4)(= 0)$ $x = -2, -4, \frac{1}{3}$	M1 M1 M1 DM1 B1, A1 [6]	M1 for simplification = 0 M1 for attempt to find a root M1 for attempt to obtain quadratic factor DM1 for obtaining linear factors or use of quadratic formula B1 for first solution A1 for the other pair

6	**(i)** $\frac{1}{2}x + 2y$	B1 B1 [2]	B1 for each term
	(ii) $y - 1$	M1 A1 [2]	M1 for difference of 2 logarithms
	(iii) $\frac{\log_8 64}{\log_8 2} + \frac{\log_8 p}{\log_8 2}$ $= 6 + 3x$	M1 B1 A1 [3]	M1 for attempt at a valid method B1 for 6 A1 for $+ 3x$
7	**(i)** $f \geqslant -3$	B1 [1]	
	(ii) $f^{-1} = \frac{\sqrt{x+3}-1}{2}$	M1 M1 A1 [3]	M1 for correct order of operations M1 for 'interchange' of x and y
	(iii) $\left(2\left(\frac{3}{1+x}\right)+1\right)^2 - 3 = 13$ $\left(\frac{7+x}{1+x}\right)^2 = 16$ $x = 1$	M1 A1 M1 B1 [4]	M1 for correct order A1 for correct simplification M1 for solution B1 for **one** solution only
8	**(a)** $2^{3-4x}\,2^{2x+8} = 2$ $3 - 4x + 2x + 8 = 1$ $x = 5$	M1 DM1 A1 [3]	M1 for to obtain powers of 2, 4 or 8 DM1 for attempt to equate powers of 2, 4 or 8, using addition
	(b) **(i)** $2\sqrt{3}$	M1 A1 [2]	M1 for attempt to obtain each term in terms of $\sqrt{3}$
	(ii) $\frac{3+\sqrt{5}\left(\sqrt{5}+2\right)}{\sqrt{5}-2\left(\sqrt{5}+2\right)}$ leading to $\frac{5\sqrt{5}+11}{1}$	M1 A1 A1 [3]	M1 for attempt to rationalise A1 for numerator A1 for denominator (can be implied)

9 **(a)** **(i)** $\frac{dy}{dx} = 5 + 4e^{-x}$	M1 A1 [2]	M1 for attempt to differentiate
(ii) when $x = 0$, $\frac{dy}{dx} = 9$		M1 for attempt to use small changes
use of $dy \approx \frac{dy}{dx} dx$ leading to	M1	
$dy \approx 9p$	A1 [2]	
(b) $\frac{dA}{dt} = 0.5$	B1	M1 for attempt to get $\frac{dA}{dx}$
$A = x^2$, $\frac{dA}{dx} = 2x$, $x = 3$	M1	DM1 for correct use of rates of change
$\frac{dx}{dt} = 0.5 \times \frac{1}{2x}$	DM1	
$= \frac{1}{12}$	A1 [4]	
10 **(i)** $\tan x = 0.25$ $x = 14.0°, 194.0°$	M1 A1,√A1 [3]	M1 for use of tan
(ii) $3 + \sin y = 3(1 - \sin^2 y)$ $3\sin^2 y + \sin y = 0$ $\sin y(3\sin y + 1) = 0$	M1	M1 for use of correct identity and attempt to simplify
$\sin y = 0$, $\sin y = -\frac{1}{3}$	DM1	DM1 for attempt to solve quadratic
$y = 180°$, $y = 199.5°, 340.5°$	B1 A1 √A1 [5]	B1 for 180° A1 for 189.5° Ft on their 189.5°
(iii) $\cos\frac{z}{3} = \frac{1}{4}$	B1	B1 for $\cos\frac{z}{3} = \frac{1}{4}$ or equivalent in terms of cos
$\frac{z}{3} = 1.3181$ leading to	M1	M1 for a correct order of operations (allow π)
$z = 3.95$ Allow 3.96, 1.25π, 1.26π	A1 [3]	

11 EITHER		
(i) $\frac{dy}{dx} = \frac{x^2(\frac{1}{x}) - 2x\ln x}{x^4}$	B3,2,1,0	–1 each error
$= \frac{1-2\ln x}{x^3}$		
when $\frac{dy}{dx} = 0$, $\ln x = \frac{1}{2}$, $x = e^{\frac{1}{2}}$, $y = \frac{\frac{1}{2}}{e}$,	M1	M1 for attempt to solve $\frac{dy}{dx} = 0$
	A1	
$y = \frac{1}{2e}$	A1	
	[6]	
(ii) $\frac{d^2y}{dx^2} = \frac{x^3(-\frac{2}{x}) - (1-2\ln x)3x^2}{x^6}$	M1	M1 for attempt at 2nd derivative
$= \frac{-5+6\ln x}{x^4}$	A1, A1	A1 for *a*, A1for *b*
	[3]	
(iii) when $x = e^{\frac{1}{2}}$, $\frac{d^2y}{dx^2}$ is –ve	M1	M1 for a correct method
$(= \frac{-2}{e^2})$, max	A1	A1 must be from correct working only
	[2]	
11 OR		
(i) $y = 3\sin(2x + \frac{\pi}{2}) (+c)$	M1, A1	M1 for $\sin(2x + \frac{\pi}{2})$
$5 = 3\sin\pi + c$, $c = 5$	M1,	M1 for attempt to find *c*
$y = 3\sin(2x + \frac{\pi}{2}) + 5$	A1	
	[4]	
(ii) $\cos(2x + \frac{\pi}{2}) = 0$	M1	M1 for attempt to solve $\frac{dy}{dx} = 0$
$x = 0, \frac{\pi}{2}, \pi$	A2,1,0	
	[3]	
(iii) when $x = \frac{3\pi}{4}$, $y = 5$	M1	M1 for attempt to obtain *y*
$\frac{dy}{dx} = 6$	M1	M1 for attempt to obtain $\frac{dy}{dx}$ and perp gradient
normal $y - 5 = -\frac{1}{6}(x - \frac{3\pi}{4})$	DM1	DM1 for attempt at straight line
$\left(y = -\frac{1}{6}x + 5.39\right)$	A1	(Must have **(i)** correct)
	[4]	